# Exercise 54

(a) Use numerical and graphical evidence to guess the value of the limit

$$\lim_{x\to 1} \frac{x^3-1}{\sqrt{x}-1}$$

(b) How close to 1 does $x$ have to be to ensure that the function in part (a) is within a distance 0.5 of its limit?

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**Solution**

Evaluate the given function at several values of $x$ to determine the limit.

$$\left.\frac{x^3-1}{\sqrt{x}-1}\right|_{x=0.9} = 5.28093$$

$$\left.\frac{x^3-1}{\sqrt{x}-1}\right|_{x=0.99} = 5.92531$$

$$\left.\frac{x^3-1}{\sqrt{x}-1}\right|_{x=0.999} = 5.9925$$

$$\left.\frac{x^3-1}{\sqrt{x}-1}\right|_{x=0.9999} = 5.99925$$

$$\left.\frac{x^3-1}{\sqrt{x}-1}\right|_{x=0.99999} = 5.99993$$

$$\left.\frac{x^3-1}{\sqrt{x}-1}\right|_{x=1.00001} = 6.00008$$

$$\left.\frac{x^3-1}{\sqrt{x}-1}\right|_{x=1.0001} = 6.00075$$

$$\left.\frac{x^3-1}{\sqrt{x}-1}\right|_{x=1.001} = 6.0075$$

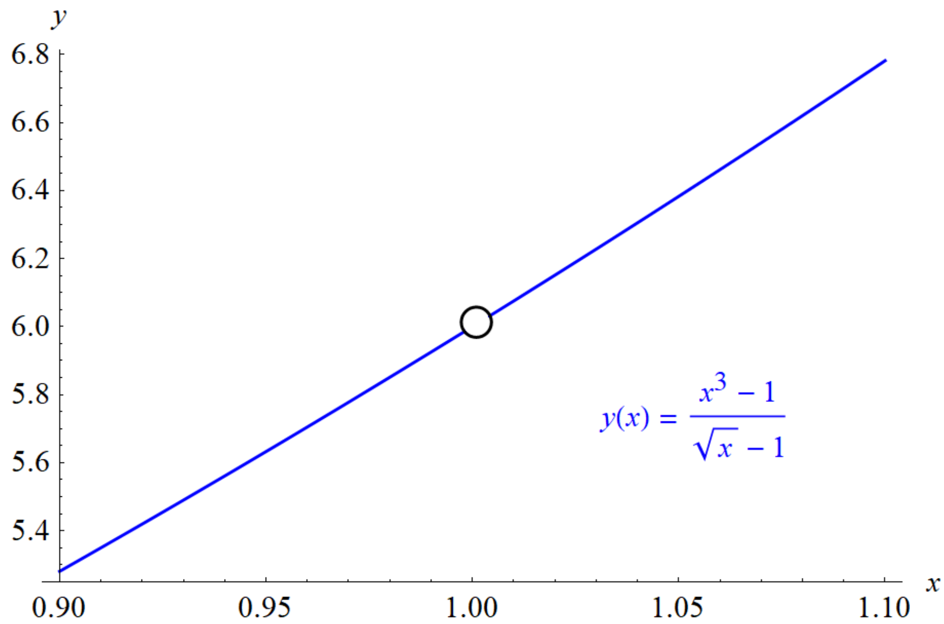
$$\left.\frac{x^3-1}{\sqrt{x}-1}\right|_{x=1.01} = 6.07531$$

$$\left.\frac{x^3-1}{\sqrt{x}-1}\right|_{x=1.1} = 6.78156$$

Therefore,

$$\lim_{x\to 1} \frac{x^3-1}{\sqrt{x}-1} = 6.$$

Below is a plot of the given function versus $x$.




This confirms the result found. Use guess-and-check to find the value of $x$ that gives 5.5 and the value of $x$ that gives 6.5.

$$\left.\frac{x^3-1}{\sqrt{x}-1}\right|_{x=0.931385} = 5.5$$

$$\left.\frac{x^3-1}{\sqrt{x}-1}\right|_{x=1.064900} = 6.5$$

Therefore, $x$ has to be within $[0.931385, 1.064900]$ for the function to be within a distance 0.5 of its limit as $x$ goes to 1.